A GPU Accelerated Convex Max-Flow Approach to Segmentation of 4-D Left-Ventricular Ultrasound

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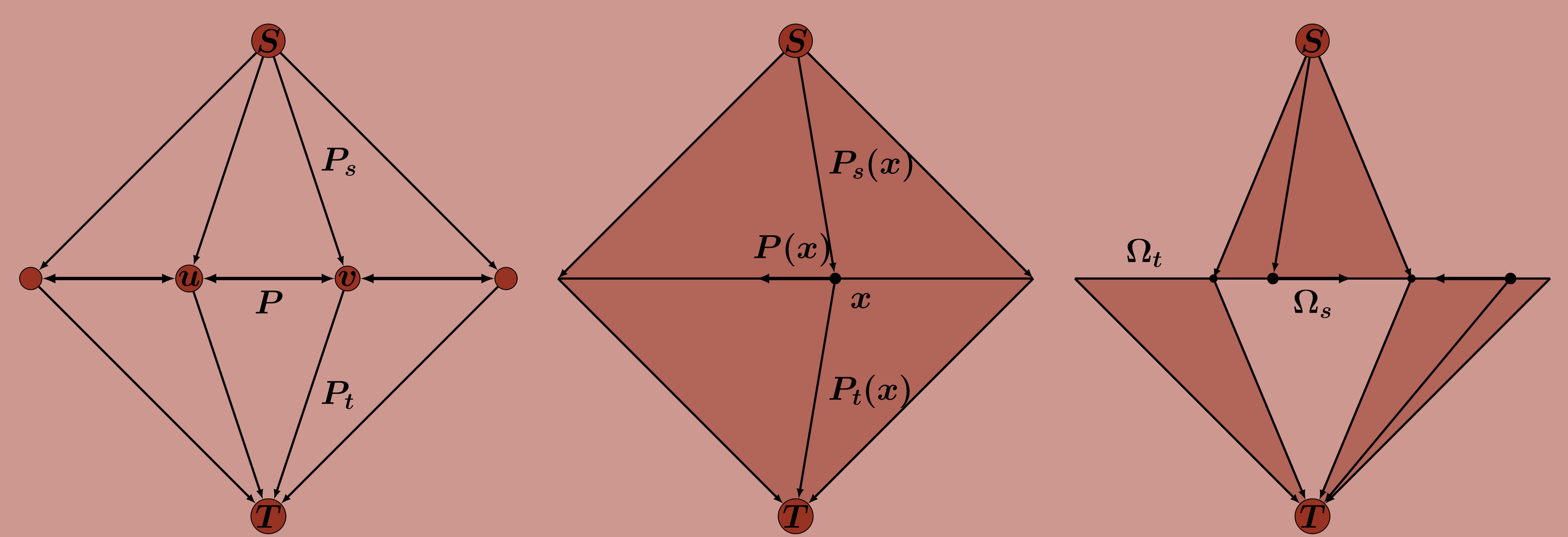
Introduction

- This study investigates the problem of partitioning an image into foreground and background using a GPU. The foreground region is modeled using a known (*a-priori* learned) statistical distribution and then segmented from its background.
- The solution to this problem was recently shown to be very useful for the following computer vision tasks:
  - Co-segmentation of image pairs;
  - Interactive image segmentation;
  - Segmentation with off-line learning;
  - Tracking.
- Studies have shown that convex relaxation approaches have a much faster convergence rate than active curve techniques that are driven by gradient-descent optimization [3].
- Unlike graph cuts, the implementation of convex relaxation approaches can be easily parallelized.
- We propose/analyze:
  - Solving a sequence of convex sub-problems;
  - A novel flow configuration approach;
  - A parallel CUDA™ implementation of the convex max-flow min-cut optimization process.

The max-flow and min-cut models

Figure : **Flow Configurations. Left:** spatially discrete (classical graph based) [2]; **Middle:** spatially continuous [4]; **Right:** The proposed spatially continuous configuration [3]. Three types of flows: the source flow $p_s(x)$ from s to x, the sink flow $p_t(x)$ from x to t, and the spatial flow $p(x)$ for each $x \in \Omega$.



Distribution Matching

- Find a region $\mathcal{R}$ whose distribution matches a known reference distribution $\mathcal{M}$.
- Evaluate the similarity between $\mathcal{P}$ and $\mathcal{M}$ using the Bhattacharyya distance:

$$\mathcal{B}(\mathcal{P}, \mathcal{M}) = \sum_{z \in \mathcal{Z}} \sqrt{\mathcal{P}(z)\mathcal{M}(z)} \quad (1)$$

  where $\mathcal{P}$ is the nonparametric estimate of the distribution within $\mathcal{R}$.
- The Bhattacharyya matching energy function is:

$$\min_{u(x)\in\{0,1\}} \left\{ E(u) := \sum_{z\in\mathcal{Z}} \left(\frac{\int_\Omega \mathcal{T}_{M,z}(x) u(x)\, dx}{\int_\Omega u(x)\, dx} \right)^{1/2} + \int_\Omega C |\nabla u(x)|\, dx \right\} \quad (2)$$

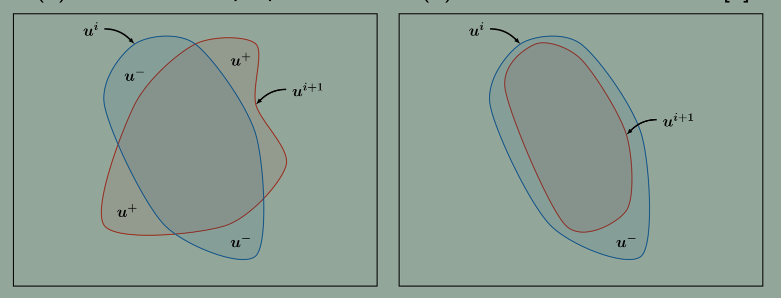
  - $\mathcal{T}_{M,z}(x) = \mathcal{K}_z(x)\mathcal{M}(z)$
  - $u(x)$ — labelling function
  - C — regularization term

Iterative bound optimization

- An iterative convex relaxation solution

$$u^{i+1} = \min_{u\in\{0,1\}} F(u, u^i), \quad i \geq 1 \ \text{s.t.} \quad (3a)$$
$$E(u) \leq F(u, u^i), \quad i \geq 1 \quad (3b)$$
$$E(u) = F(u, u) \quad \forall u : \Omega \rightarrow \{0, 1\} \quad (3c)$$

Figure : **Iterative bound optimization:** (a) Illustrates the proposed bound; (b) Illustrates the bound in [1]. u^i denotes the labelling at iteration i;



(a) (b)

The continuous max-flow model

The continuous max-flow model can be formulated by:

$$\max_{p_s, p_t, p} \int_{\Omega_s} p_s \, dx \quad (4)$$

constrained by the following flow capacities:

$$p_s(x) \leq C_v^i(x); \quad p_t(x) \leq C_w^i(x); \quad |p(x)| \leq C \quad (5)$$

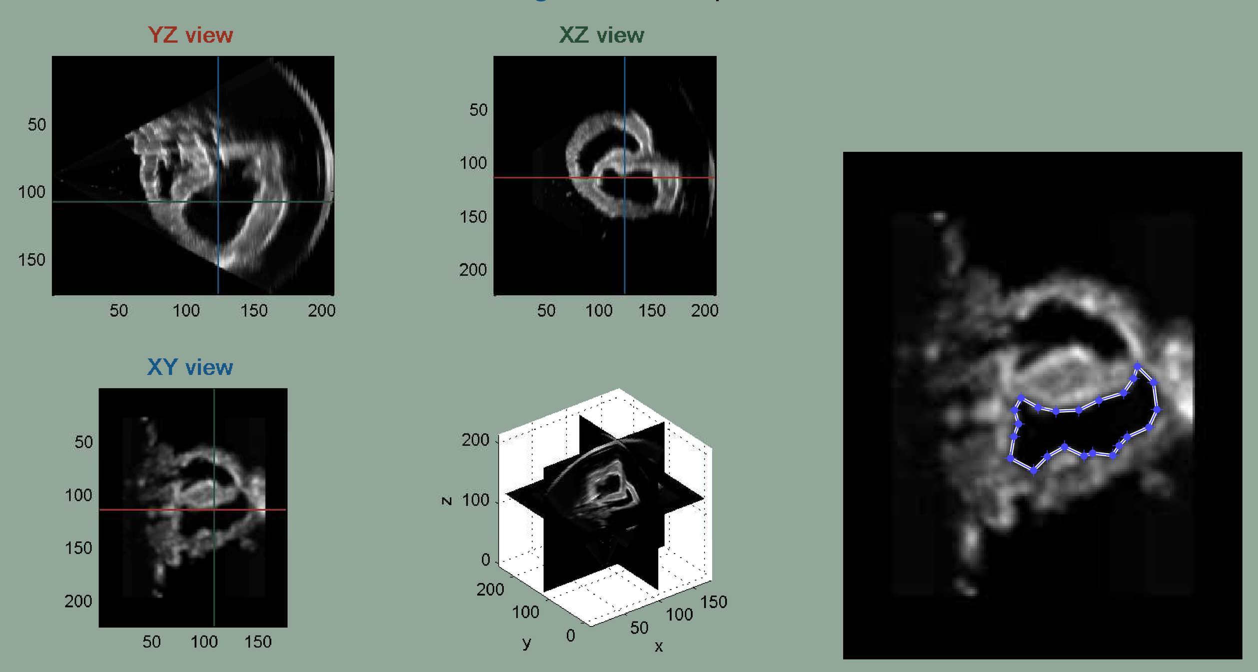
and flow conservation conditions:

$$-p_s(x) + \operatorname{div} p(x) = 0; \quad p_t(x) + \operatorname{div} p(x) = 0 \quad (6)$$

Application to Left-Ventricular (LV) Segmentation

- We used two components in our algorithms:
  - Intensity matching term
  - Distance matching term
- The density is estimated from these two components with a 2-dimensional histogram consisting of 256 × 256 bins.
- The reference distribution is learned from a manual segmentation on a single 2D slice provided by the user.
- Input Data:
  - Five 3D volumes are acquired from a Philips iE33 ultrasound scanner.
  - Each volume consists of 224 × 176 × 208 voxels.

Figure : User input



(a) An approximate center (b) Initial segmentation on a single 2D slice

Results

Figure : Extracted 3D surface of LV Data using the proposed algorithm.

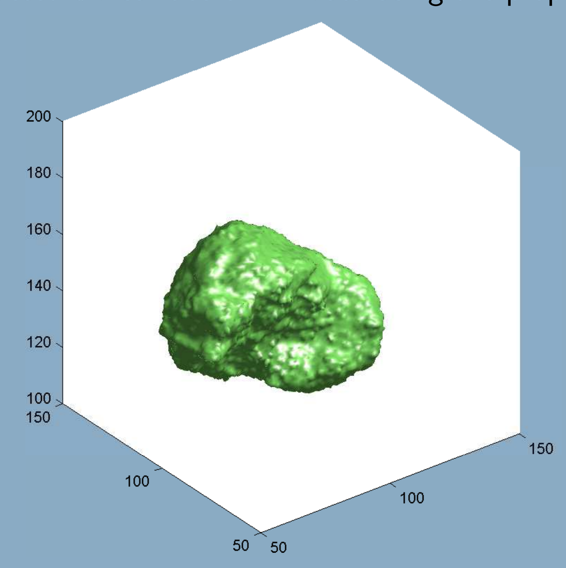


Figure : The segmentation results on 2D-slices

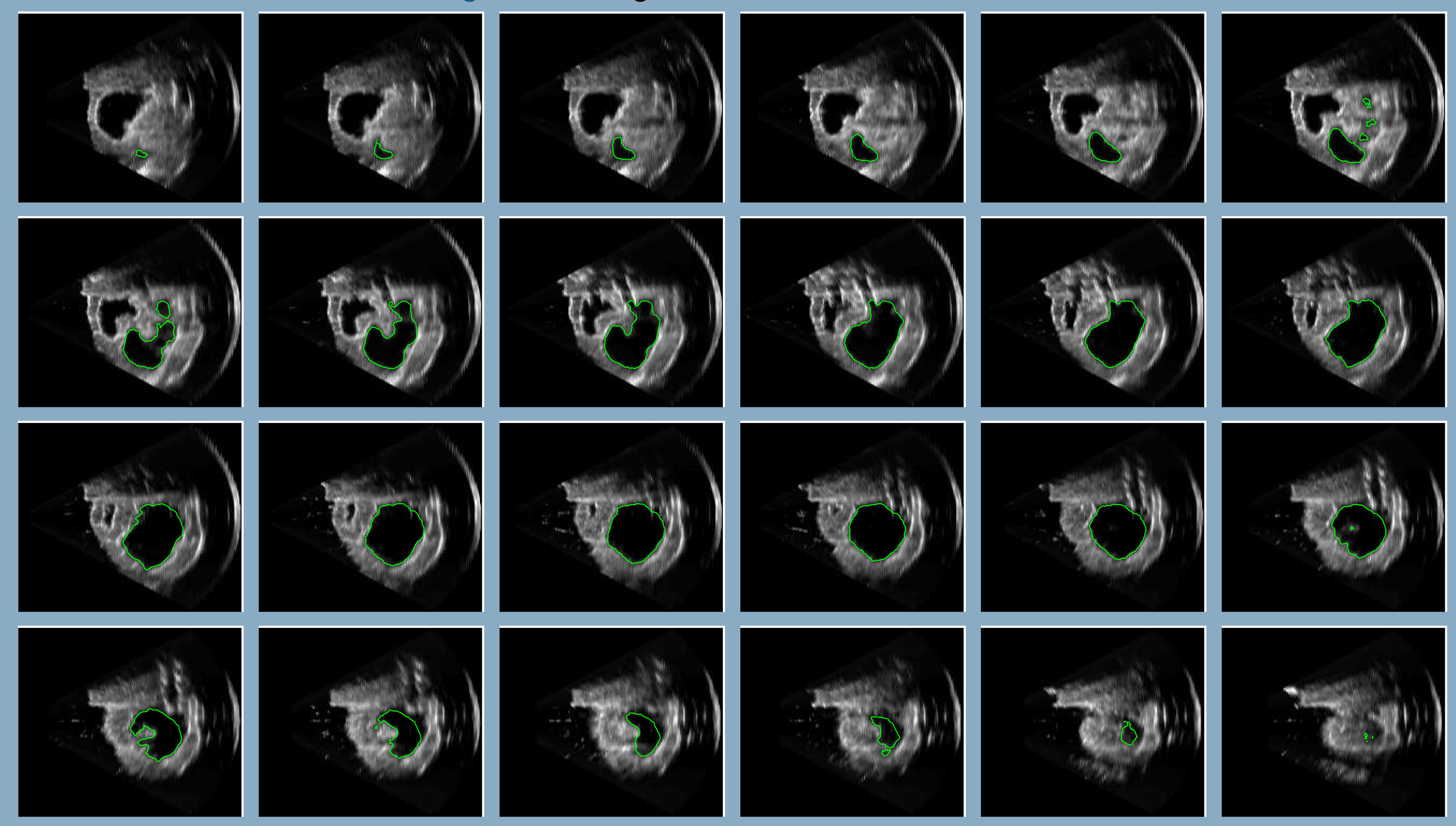


Table : Computational times of the GPU/CPU implementations of the algorithm.

Implementation	3D (224×176×208)
GPU	0.79 ± 0.04 seconds
CPU	26.85 ± 0.88 seconds

- GPU implementation
  - We used two NVIDIA Tesla™ C2070 Computing Processors to test the GPU implementation of the algorithm
  - The GPU implementation yielded a $34\times$ speed-up in comparisons to CPU implementation of the algorithm
  - Both GPU and CPU versions were implemented in C and Matlab™.

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